which is easily verified to agree with the previous result. The corresponding double root is then found to be

$$x_0 = \frac{c_0^{1/3}}{2}[(a+b)^{2/3} - (a-b)^{2/3}] = \sqrt{\frac{a^2+b^2-c_0^2}{3}} = \frac{6abc_0}{a^2+b^2+8c_0^2}. \tag{11}$$

The remaining two roots are obtained by adding to the negative of the double root

$$\pm\sqrt{\frac{a^2+b^2+8c_0^2}{3}}. \tag{12}$$

The maximum value of $f(x)$ at $x = (x_0/c_0)c$ will be found to be

$$2\left[1 - \frac{c_0^2}{c^2}\right], \tag{13}$$

and it gives the same results as the discriminant and in a much more evident form. This shows that the first two positive roots are separated by $(x_0/c_0)c$.

In Osgood's *Calculus*, page 404, is given a solution of the case $a = 15$, $b = 10$, $c = 1$. Here $c_0 = 13.741$, $x_0 = 6.738$, and since $a^2/b^2 = 2.25 > 6\sqrt{3} - 9$ this example falls in II2 and there is only one inscribed rectangle. If $c = 13$ there is no inscribed rectangle. If $c = 14$ there are two inscribed rectangles, one having the side 5.+ and the other the side 8.+.

II. Note on the Preceding by H. P. Manning, Brown University.

There is no particular distinction between the sides c and x of the inscribed rectangle. We can call them x and y, and equation (3) or (3′) will then be represented by a curve of the fourth degree in which many of the results of the above discussion appear graphically.

Moreover, m and n satisfy the equation $m^2 - n^2 = am - bn$, and so are represented by the points of an equilateral hyperbola, which passes through the vertices of the given rectangle if we lay off m and n from its lower left-hand corner.

273 (Number Theory) [1917, 427]. Proposed by V. M. SPUNAR, Chicago, Ill.

The ratio of the chances that all numbers ending in 1 or 9 and those ending in 3 or 7 are composite is 3 : 2[1].

Note by Norman Anning, University of Maine.

Since

$$1\times 1 \equiv 3\times 7 \equiv 9\times 9 \equiv 1 \pmod{10},\qquad 1\times 3 \equiv 7\times 9 \equiv 3 \pmod{10},$$
$$1\times 7 \equiv 3\times 9 \equiv 7 \pmod{10},\qquad 1\times 9 \equiv 3\times 3 \equiv 7\times 7 \equiv 9 \pmod{10},$$

the conclusion might be drawn that in the long run as many primes end in 1 as in 9 and as many end in 3 as in 7 and that there would be more of the latter than of the former. A census of primes taken over a considerable range supports these statements but does not point towards the ratio "3 : 2." Counting cases up to 3,200, a number chosen at random, shows the following results:

110 primes end in 1, 113 in 3, 116 in 7 and 111 in 9.

Since these numbers are so nearly in equilibrium and since primes are so perfectly lawless no statement could be hazarded about the distribution in a larger interval.

2728 [1918, 397]. Proposed by NORMAN ANNING, University of Maine.

A material triangle of uniform density and thickness is of such a shape that when suspended from the vertices in succession, the lower sides have slopes of 1 : 1, $1\frac{1}{2}$: 1, and 3 : 1. Construct the triangle given that the shortest side is 10 inches.

By definition, an a : 1 slope makes an angle with the vertical whose tangent is a.

[1] The enunciation of the problem is not clear. The chance that all numbers ending in 1 or 9 are composite numbers is zero. — Editors.